数理解析研究所講究録
751巻 1991年 13-27

KMS states for self-dual CCR algebras

新潟大 自然科学 荻 秀和 (Hidekazu Ogi)

1. Introduction

A dynamical system is a pair consisting of a $*$-algebra $\mathfrak{A}$ and a one-parameter automorphism group $\{\tau_t\}_{t\in\mathbb{R}}$ of $\mathfrak{A}$. In the following, we assume that $\mathfrak{A}$ is unital ($\mathbb{1} \in \mathfrak{A}$).

A linear functional φ on $\mathfrak{A}$ is called a state if φ satisfies

$$\varphi(\mathbb{1}) = 1 \quad \text{and} \quad \varphi(A^*A) \geq 0 \quad \text{for all} \quad A \in \mathfrak{A}.$$

A state φ on $\mathfrak{A}$ is called a β-KMS state for $\{\tau_t\}_{t\in\mathbb{R}}$ (or simply a KMS state) if it satisfies the following KMS-condition (at the inverse temperature β, where β is a positive number):

For every pair of elements A and B of $\mathfrak{A}$, there exists a function F defined on the closure $\overline{D}_\beta$ of the strip $D_\beta = \{z \in \mathbb{C} : 0 < \mathrm{Im} z < \beta\}$ such that (1) F is bounded and continuous on $\overline{D}_\beta$, (2) F is analytic on D_β, and (3) the boundary values of F are related to φ by

$$F(t) = \varphi(A\tau_t(B)), \; F(t + i\beta) = \varphi(\tau_t(B)A) \quad (t \in \mathbb{R}).$$

In statistical mechanics, τ_t describes the time evolution of the system and the KMS-condition is considered to be a condition characterizing the state of the system at thermal equilibrium (with the absolute temperature $T = (k\beta)^{-1}$ where k is the Boltzmann constant).

In the theory of bounded operator algebras, it is well-known that KMS states play an important role for a study of structures of von Neumann algebras. If we represent a CCR algebra as an operator algebra in a Hilbert space, then it is always unbounded. To avoid the difficulty coming from the unboundedness we usually consider a bounded operator algebra whose generators satisfy the Weyl-Segal commutation relations, but we can not directly observe the annihilation and creation operators in it. Thus it seems meaningful to study the unbounded operator algebra as itself and to investigate KMS states on it.

Let $\mathfrak{A}(K,r,\Gamma)$ be the self-dual CCR algebra over a triplet (K,r,Γ), where K is assumed to be a Hilbert space, and let $\{\tau_t\}_{t\in\mathbb{R}}$ be a Bogoliubov automorphism group of $\mathfrak{A}(K,r,\Gamma)$ induced by a strongly continuous one-parameter unitary group on K. In this paper, we introduce some continuity for linear functionals on $\mathfrak{A}(K,r,\Gamma)$ and, under this continuity, we study the structure of KMS states for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$. In section 2, we recall the definitions of a self-dual CCR algebra $\mathfrak{A}(K,r,\Gamma)$, a Bogoliubov automorphism of $\mathfrak{A}(K,r,\Gamma)$, and a quasifree state on $\mathfrak{A}(K,r,\Gamma)$. In section 3, we introduce some continuity for linear functionals on $\mathfrak{A}(K,r,\Gamma)$. Under this continuity, we study the existence of KMS states for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$. In section 4, under the continuity introduced in section 3, we study the uniqueness and the non-uniqueness of KMS states for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$.

2. Preliminaries

In this section, we recall the definitions of a self-dual CCR algebra $\mathfrak{A}(K,r,\Gamma)$, a Bogoliubov automorphism of $\mathfrak{A}(K,r,\Gamma)$, and a quasifree state on $\mathfrak{A}(K,r,\Gamma)$ (cf. [1]).

Let K be a complex linear space. Let $r(\cdot,\cdot)$ be a hermitian form on $K \times K$ and Γ be an anti-linear operator on K such that

$$\Gamma^2 = 1 \quad \text{and} \quad r(\Gamma f, \Gamma g) = - r(g, f) \quad \text{for} \quad f, g \in K.$$

(K,r,Γ) is called the basic triplet. A self-dual CCR algebra $\mathfrak{A}(K,r,\Gamma)$ over (K,r,Γ) is a $*$-algebra generated by an identity $\mathbb{1}$ and $\{B(f) : f \in K\}$ such that

(1) $B(f)$ is complex linear in f,

(2) $B(f)^* = B(\Gamma f)$,

(3) $B(f)^*B(g) - B(g)B(f)^* = r(f, g)\mathbb{1}$ for $f, g \in K$.

Let (K',r',Γ') be an another basic triplet. If a linear operator $U : K \longrightarrow K'$ satisfies

$$U\Gamma = \Gamma'U \quad \text{and} \quad r'(Uf, Ug) = r(f, g) \quad \text{for} \quad f, g \in K,$$

then we can define a $*$-homomorphism τ_U from $\mathfrak{A}(K,r,\Gamma)$ into $\mathfrak{A}(K',r',\Gamma')$ by

$$\tau_U(B(f)) = B(Uf) \quad \text{for} \quad f \in K.$$

If U is a bijective linear operator on K satisfying the above conditions, then it is called a Bogoliubov transformation on K and the associated $*$-automorphism τ_U of $\mathfrak{A}(K,r,\Gamma)$ is called a Bogoliubov automorphism.

Next we define quasifree states on $\mathfrak{A}(K,r,\Gamma)$.

Definition 2.1. A state φ on $\mathfrak{A}(K,r,\Gamma)$ is called a quasifree state if φ satisfies the following conditions; for every $n = 1,2,\cdots$,

$$\varphi(B(f_1)\cdots B(f_{2n-1})) = 0$$

$$\varphi(B(f_1)\cdots B(f_{2n})) = \sum \prod_{j=1}^{n} \varphi(B(f_{s(j)})B(f_{s(j+n)})),$$

where the sum is over all permutations s satisfying $s(1) < s(2) < \cdots < s(n)$, $s(j) < s(j+n)$, $j = 1,2,\cdots,n$.

3. Existence of KMS states

In this section, we introduce some continuity for linear functionals on $\mathfrak{A}(K,r,\Gamma)$ and, under this continuity, we study the existence of KMS states for $\{\mathfrak{A}(K,r,\Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$.

Throughout the rest of this paper, we suppose the following situations (*):

(i) (K, r, Γ) is a basic triplet such that
K is a Hilbert space under an inner product $(\cdot,\cdot)$ and
Γ is an anti-unitary operator on K.

(ii) H is a self-adjoint operator in K such that
$U_t = \exp(itH)$ is a Bogoliubov transformation for (K, r, Γ).

Denote τ_t the Bogoliubov automorphism of $\mathfrak{A}(K, r, \Gamma)$ induced by U_t.

For the dynamical system $\{\mathfrak{A}(K, r, \Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$ given as above, we study the existence of KMS states under the following continuity.

Definition 3.1. A state φ on $\mathfrak{A}(K, r, \Gamma)$ is said to be K-continuous if, for every $n = 1, 2, \cdots$, the mapping

$$(f_1, \cdots, f_n) \in K \times \cdots \times K \longrightarrow \varphi(B(f_1)\cdots B(f_2))$$

is continuous, where $K \times \cdots \times K$ is the n-fold product space of the Hilbert space K.

Theorem 3.2. Under the situations (*), there exists a K-continuous β-KMS state for $\{\tau_t\}_{t\in\mathbb{R}}$ on the self-dual CCR algebra $\mathfrak{A}(K, r, \Gamma)$ $(\beta \in (0, +\infty))$ if the following equality holds;

$$r(f, g) = (f, (e^{\beta H}-1)(e^{\beta H}+1)^{-1}g) \quad \text{for} \quad f, g \in K.$$

Proof. Assume that

$$r(f, g) = (f, (e^{\beta H}-1)(e^{\beta H}+1)^{-1}g) \quad \text{for} \quad f, g \in K \quad \text{and} \quad \beta \in (0, +\infty).$$

And we set

$$S = e^{\beta H}(e^{\beta H}+1)^{-1}.$$

Then, by the assumption, there exists a quasifree state φ which is given by

$$\varphi(B(f)^{*}B(g)) = (f, Sg) \quad \text{for} \quad f, g \in K.$$

It follows from the definition of S that

$$0 \le S \le 1 \quad \text{and} \quad \Gamma S \Gamma = 1 - S.$$

Since Γ is an anti-unitary operator on K and S does not have an eigenvalue 0, S also does not have an eigenvalue 1. Thus, if we set

$$\mathcal{D} = \mathcal{D}(S^{-1/2}) \cap \mathcal{D}((1-S)^{-1/2}),$$

then $\mathcal{D}$ is a Γ-invariant dense subspace of K. We put

$$U_t = \exp(itH).$$

Then we have

$$\Gamma U_t \Gamma = U_t.$$

For g in $\mathscr{D}$, we remark that $U_t g$ has a extension $U_z g$ which is strongly continuous for $-\frac{\beta}{2} \le \mathrm{Im} z \le \frac{\beta}{2}$ and strongly analytic for $-\frac{\beta}{2} < \mathrm{Im} z < \frac{\beta}{2}$. Since

$$\varphi(B(f)\tau_t(B(g))) = \varphi(B(\Gamma f)^* B(U_t g)) = (\Gamma f,\ SU_t g),$$

we get its continuous extension $(\Gamma f,\ SU_z g)$ for $-\frac{\beta}{2} \le \mathrm{Im} z \le \frac{\beta}{2}$ whicl is analytic in its interior. Remark that $\varphi(\tau_t(B(g))B(f))$ also has ε continuous extension $(\Gamma U_z g,\ Sf)$ on the same closed strip which is analytic in its interior. Further we have

$$(\Gamma f,\ SU_{t+i\beta/2} g) = (\Gamma f,\ SS^{-1/2}(1-S)^{1/2} U_t g)$$

$$= (\Gamma S^{1/2}(1-S)^{1/2} U_t g,\ f) = (\Gamma S^{1/2}(1-S)^{-1/2} U_t g,\ Sf)$$

$$= (\Gamma U_{t-i\beta/2} g,\ Sf),$$

where we used the equality $\Gamma S^{-1}\Gamma = (1-S)^{-1}$. Thus we get the desired continuous function $F_{B(f),B(g)}$ which appeared in the KMS-condition for f in K and g in $\mathscr{D}$. Using the density of $\mathscr{D}$ in K, we can also get the function $F_{B(f),B(g)}$ for every f, g in K.

For general elements A, B in $\mathfrak{A}(K,r,\Gamma)$, using the definition of quasifreeness, we can show the existence of the function $F_{A,B}$ ir the KMS-condition. This completes the proof.

Remark 3.3. Let ℓ be a pre-Hilbert space under an inner product $(\cdot,\cdot)$, and let L be the completion of ℓ. Let L^* be the conjugate Hilbert space of L, and let Γ' be a bijective anti-linear operator from L onto L^* defined by $\Gamma' f = f$. Take a self-adjoint operator h in L such that ℓ is globally invariant under a one-parameter unitary group $V_t = \exp(ith)$. We set

$$K = L \oplus \Gamma' L, \quad \Gamma = \begin{pmatrix} 0 & \Gamma'^{-1} \\ \Gamma' & 0 \end{pmatrix}, \quad H = \begin{pmatrix} h & 0 \\ 0 & -\Gamma' h \Gamma'^{-1} \end{pmatrix},$$

and

$$r(f, g) = (f, (e^{\beta H}-1)(e^{\beta H}+1)^{-1} g) \quad (f, g \in K \text{ and } \beta \in (0, +\infty)).$$

Then we can easily check that (K, r, Γ) and H satisfy the situations (*).

A CCR algebra $\mathfrak{A}_{CCR}(\ell)$ over ℓ is a *-algebra generated by an identity and $\{a(f), a^*(f) : f \in \ell\}$ such that

(1) $a^*(f)$ is complex linear in f,

(2) $a(f)^* = a^*(f)$,

(3) $[a(f), a^*(g)] = (f, g)$,
$[a^*(f), a^*(g)] = [a(f), a(g)] = 0$ for $f, g \in \ell$,

where $[A, B] = AB - BA$. Suppose that h does not have an eigenvalue 0 and $\mathscr{D}((e^{\beta h}+1)^{1/2}(e^{\beta h}-1)^{-1/2}) \supset \ell$. Put $\ell' = \mathscr{D}((e^{\beta h}+1)^{1/2}(e^{\beta h}-1)^{-1/2})$. Let Q be a operator on K defined by

$$Q(f_1 \oplus \Gamma' f_2) = (e^{\beta h}-1)^{1/2}(e^{\beta h}+1)^{-1/2} f_1 \oplus 0 \quad \text{for} \quad f_1,\ f_2 \in L.$$

Then we can give a $*$-isomorphism Φ between $\mathfrak{A}(K,r,\Gamma)$ and $\mathfrak{A}_{CCR}(\ell')$ $(\supset \mathfrak{A}_{CCR}(\ell))$ by

$$\Phi(B(f)) = a^*(Qf) + a(Q\Gamma f) \quad \text{for} \quad f \in K.$$

Let $\{\sigma_t\}_{t\in\mathbb{R}}$ (resp. $\{\sigma'_t\}_{t\in\mathbb{R}}$) be a one-parameter group of $*$-automorphisms of $\mathfrak{A}_{CCR}(\ell)$ (resp. $\mathfrak{A}_{CCR}(\ell')$) induced by $V_t = \exp(ith)$, and let $\{\tau_t\}_{t\in\mathbb{R}}$ be a Bogoliubov automorphism group of $\mathfrak{A}(K,r,\Gamma)$ induced by $U_t = \exp(itH)$. Then $\Phi \circ \tau_t = \sigma'_t \circ \Phi$. Therefore the dynamical system $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$ is equivalent to the dynamical system $\{\mathfrak{A}_{CCR}(\ell'),\ \{\sigma'_t\}_{t\in\mathbb{R}}\}$ $(\supset \{\mathfrak{A}_{CCR}(\ell),\ \{\sigma_t\}_{t\in\mathbb{R}}\})$.

4. Uniqueness and non-uniqueness of KMS states

In this section, under the K-continuity defined in section 3, we study the uniqueness and the non-uniqueness of KMS states for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$.

Lemma 4.1. Under the situations (*), let φ be a K-continuous β-KMS state for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$. Then we have, for any $A \in \mathfrak{A}(K,r,\Gamma)$ and $f \in \mathscr{D}(He^{-\beta H})$,

$$\varphi(B(f)A) = \varphi(AB(e^{-\beta H}f)).$$

Proof. We define the function G on $\overline{D}_\beta$ by

$$G(z) = \varphi(AB(e^{izH}f)),$$

where $\overline{D}_\beta$ is the closure of the strip $D_\beta = \{z \in \mathbb{C} : 0 < \mathrm{Im} z < \beta\}$. Since φ is K-continuous, G is continuous on $\overline{D}_\beta$ and analytic on D_β. Let F be a function satisfying the KMS-condition for A and $B(f)$. Since $G(t) = F(t)$ $(t \in \mathbb{R})$, by [3, Proposition 5.3.6], we have

$$G(z) = F(z) \quad \text{for all} \quad z \in \overline{D}_\beta.$$

Therefore we have

$$\varphi(AB(e^{-\beta H}f)) = G(i\beta) = F(i\beta) = \varphi(B(f)A).$$

This completes the proof.

Proposition 4.2. Under the situations (*), let φ be a K-continuous β-KMS state for $\{\mathfrak{A}(K,r,\Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$. If H does not have an eigenvalue 0, then φ is quasifree.

Proof. By Lemma 4.1, we have, for $f \in \mathcal{D}(He^{-\beta H})$ and $g \in K$,

$$\varphi(B(f)B(g)) = \varphi(B(g)B(e^{-\beta H}f)) = r(\Gamma g,\ e^{-\beta H}f) + \varphi(B(e^{-\beta H}f)B(g)).$$

Hence we have

$$\varphi(B((1-e^{-\beta H})f)B(g)) = r(\Gamma g,\ e^{-\beta H}f).$$

Let h be in $\mathscr{D}(He^{-\beta H}(1-e^{-\beta H})^{-1})$ and put $f = (1-e^{-\beta H})^{-1}h$. Then $f \in \mathscr{D}(He^{-\beta H})$ and so

$$\varphi(B(h)B(g)) = r(\Gamma g,\ e^{-\beta H}(1-e^{-\beta H})^{-1}h).$$

Analogously, we have, for $f_1 \in \mathscr{D}(He^{-\beta H})$,

$$\varphi(B(f_1)\cdots B(f_{2n})) = \varphi(B(f_2)\cdots B(f_{2n})B(e^{-\beta H}f_1))$$

$$= \sum_{p=2}^{2n} r(\Gamma f_p,\ e^{-\beta H}f_1)\varphi(B(f_2)\cdot\cdot B(f_{p-1})B(f_{p+1})\cdot\cdot B(f_{2n}))$$

$$+\ \varphi(B(e^{-\beta H}f_1)B(f_2)\cdots B(f_{2n})).$$

Hence by the replacement of f_1 by $(1-e^{-\beta H})^{-1}h_1$ ($h_1 \in \mathscr{D}(He^{-\beta H}(1-e^{-\beta H})^{-1})$), we have

$$\varphi(B(h_1)B(f_2)\cdots B(f_{2n}))$$

$$= \sum_{p=2}^{2n} r(\Gamma f_p,\ e^{-\beta H}(1-e^{-\beta H})^{-1}h_1)\varphi(B(f_2)\cdot\cdot B(f_{p-1})B(f_{p+1})\cdot\cdot B(f_{2n}))$$

$$= \sum_{p=2}^{2n} \varphi(B(h_1)B(f_p))\varphi(B(f_2)\cdot\cdot B(f_{p-1})B(f_{p+1})\cdot\cdot B(f_{2n})).$$

Thus by induction, we have, for $h_i \in \mathcal{D}(He^{-\beta H}(1-e^{-\beta H})^{-1})$ (i = 1,2,$\cdots$,2n),

$$\varphi(B(h_1)B(h_2)\cdots B(h_{2n})) = \sum \prod_{j=1}^{n} \varphi(B(h_{s(j)})B(h_{s(j+n)})),$$

where the sum is over all permutations s satisfying $s(1) < s(2) < \cdots < s(n)$, $s(j) < s(j+n)$, $j = 1,2,\cdots,n$. Since φ is K-continuous, the above equality holds for all h_i in K.

If the number of $B(f_i)$ is odd, then we show that the corresponding value of φ is zero. By the previous discussion, if the number is odd, then $\varphi(B(f_1)\cdots B(f_{2n-1}))$ is expressed by the sum of products of numbers with factors $\varphi(B(f_i)B(f_j))$ and $\varphi(B(f_k))$. Therefore it is enough to show that $\varphi(B(f)) = 0$ for $f \in K$. For $f \in \mathcal{D}(He^{-\beta H})$, we have

$$\varphi(B(f)) = \varphi(B(f)\mathbb{1}) = \varphi(\mathbb{1}B(e^{-\beta H}f)) = \varphi(B(e^{-\beta H}f)).$$

Hence by the replacement of f by $(1-e^{-\beta H})^{-1}h$ ($h \in \mathcal{D}(He^{-\beta H}(1-e^{-\beta H})^{-1})$), we have $\varphi(B(h)) = 0$. Since φ is K-continuous, we have $\varphi(B(f)) = 0$ for $f \in K$. Therefore φ is quasifree. This completes the proof.

We are now in a position to prove the uniqueness of K-continuous β-KMS states for $\{\mathfrak{A}(K,r,\Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$.

Theorem 4.3. Under the situations (*), if H does not have an eigenvalue 0 and there exists a K-continuous β-KMS state for $\{\mathfrak{A}(K,r,\Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$, it is unique.

Proof. Let φ be a K-continuous β-KMS state for $\{\mathfrak{A}(K,r,\Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$. Then it follows from Proposition 4.2 that it is quasifree. Thus the value $\varphi(A)$ for A in $\mathfrak{A}(K,r,\Gamma)$ is determined by the two point function of the form $\varphi(B(f)B(g))$. The first part of the proof of Proposition 4.2 shows that, for $f \in \mathfrak{D}(He^{-\beta H}(1-e^{-\beta H})^{-1})$ and $g \in K$,

$$\varphi(B(f)B(g)) = r(\Gamma g, e^{-\beta H}(1-e^{-\beta H})^{-1}f).$$

The K-continuity of φ and the density of $\mathfrak{D}(He^{-\beta H}(1-e^{-\beta H})^{-1})$ in K prove the theorem.

Corollary 4.4. Under the situations (*), assume that the following equality holds;

$$r(f, g) = (f, (e^{\beta H}-1)(e^{\beta H}+1)^{-1}g) \quad \text{for } f, g \in K \text{ and } \beta \in (0, +\infty).$$

If H does not have an eigenvalue 0, then there exists a unique K-continuous β-KMS state for $\{\mathfrak{A}(K,r,\Gamma), \{\tau_t\}_{t\in\mathbb{R}}\}$.

Proof. Combining Theorem 3.2 and Theorem 4.3, we get the conclusion.

Next, we will consider the case that H has an eigenvalue 0 in the above corollary.

Theorem 4.5. Under the situations (*), assume that the following equality holds;

$$r(f,\ g) = (f,\ (e^{\beta H}-1)(e^{\beta H}+1)^{-1}g) \quad \text{for} \quad f,\ g \in K \quad \text{and} \quad \beta \in (0,\ +\infty).$$

If H has an eigenvalue 0, then there exist many K-continuous β-KMS states for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$.

Proof. Let P denote the projection in K whose range is the eigenspace of H corresponding to 0. Take an arbitrary positive bounded operator T on K with $PTP = T$ and $\Gamma T = T\Gamma$, and set

$$\rho(B(f)^*B(g)) = (f,\ e^{\beta H}(e^{\beta H}+1)^{-1}g) + (f,\ Tg) \quad \text{for} \quad f,\ g \in K.$$

Then we have

$$\rho(B(f)^*B(g)) - \rho(B(g)B(f)^*) = r(f,\ g) \quad \text{for} \quad f,\ g \in K.$$

Hence ρ can be extended to a quasifree state on $\mathfrak{A}(K,r,\Gamma)$, which is denoted by the same notation. It follows from $PTP = T$ that

$$(e^{itH}f,\ Tg) = (f,\ Tg) = (f,\ Te^{itH}g) \quad \text{for} \quad f,\ g \in K \quad \text{and} \quad t \in \mathbb{R}.$$

As in the proof of Theorem 3.2, we can prove that ρ is a K-continuous β-KMS state for $\{\mathfrak{A}(K,r,\Gamma),\ \{\tau_t\}_{t\in\mathbb{R}}\}$. It is clear that there exist many choices of the above T. This completes the proof.

References

[1] H. Araki and M. Siraishi, On quasifree states of the canonical commutation relations (I), Publ. RIMS Kyoto Univ., 7(1971/1972), 105-120.

[2] H. Araki, On quasifree states of the canonical commutation relations (II), Publ. RIMS Kyoto Univ., 7(1971/1972), 121-152.

[3] O. Bratteli and D. W. Robinson, Operator algebras and quantum statistical mechanics II, Springer-Verlag, Berlin-Heidelberg-New York, 1981.

[4] A. Inoue, An unbounded generalization of the Tomita-Takesaki theory, Publ. RIMS Kyoto Univ., 22(1986), 725-765.

[5] R. V. Kadison and J. R. Ringrose, Fundamentals of the theory of operator algebras II, Academic press, New York-London, 1986.

[6] H. Kurose and H. Ogi, On a generalization of the Tomita-Takesaki theorem for a quasifree state on a self-dual CCR algebra, Nihonkai Math. J., 1(1990), 19-42.

[7] R. T. Powers, Self-adjoint algebras of unbounded operators, Comm. Math. Phys., 21(1971), 85-124.